Plunger Lift Analysis, Troubleshooting and Optimization

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ALRDC Section 2.4e - Guidelines & Recommended Practices for use of Tubing Plunger Lift systems, discusses the considerations, applications, costs and recommended practices.

Where the document will reside?

http://www.alrdc.com/recommendations/Gas%20Well%20Deliquification/index.htm

Or go to www.alrdc.com click menu “Services – Open to All” click “Recommended Practices” next “Gas Well Deliquification” under hotlinks “Artificial Lift Selection” scroll down to section 2.4e Tubing Plungers and click on Section 2.4e --- Tubing Plungers
Note: Master valve & lubricator catcher assembly must the appropriate size ID for the size tubing installed in the well. Changes in ID through the tubing, master valve, and lubricator/catcher should be negligible.

Flanged master valves are recommended to eliminate potential of valve breakage.
Plunger Cycle

Plunger lift operation cycle can be divided into three parts:

1) **Shut-in**: Surface valve closed, flow shut-in, plunger falls down the tubing. Goal of the operator or controller is to try to achieve Shut-in of the well for the shortest amount of time possible, but long enough for plunger to reach bottom. And long enough for the pressure to build high enough to bring the plunger back to surface.

   *How long does it take plunger to get to bottom during shut-in?*

2) **Unloading**: Surface valve open and pressure stored in the casing lifts the accumulated liquid and plunger to the surface.

   *How much Casing Pressure is Required?*

3) **After-flow**: Surface valve open and well continues to flow after plunger reaches the surface. Plunger held at surface by differential pressure from flow of gas up the tubing. Well is producing gas. Most liquid produced from the formation tends to fall back, accumulating at the bottom of the tubing. The goal of the operator or controller is to Flow the well only until the well begins to load with liquids.

Thanks: Dan Phillips and Scott Listiak
How: Listen to Plunger Signals During Shut-in

1) 3 Channel High Frequency (30Hz or greater) Data Acquisition

2) Tubing
   a) Pressure
   b) Acoustic signal

3) Casing pressure

Just Listen To Plunger
Equipment on Well
Conventional Plunger Cycle

[A] Valve Closes, Shut-in Begins and Pressure Starts Increasing

[2] Shut-in Valve Closed, w/ Pressure Increasing

[B] Valve Opens, Unloading Begins


[C] Valve Closes, Cycle Repeats
Pressures During Normal Well Cycle

500,000 Data Points Collected in 90 Min.

<table>
<thead>
<tr>
<th>Plunger Cycle Events</th>
<th>Elapsed Time</th>
<th>tubing Pressure (psi)</th>
<th>Casing Pressure (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[A] Valve Closes (Shut In Begins)</td>
<td>2.267</td>
<td>137.0</td>
<td>192.7</td>
</tr>
<tr>
<td>[1] Plunger Hits Liquid</td>
<td>47.467</td>
<td>235.5</td>
<td>273.1</td>
</tr>
<tr>
<td>[2] Plunger On Bottom</td>
<td>59.283</td>
<td>235.5</td>
<td>290.7</td>
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<tr>
<td>[B] Valve Opens (Unloading Begins)</td>
<td>69.267</td>
<td>272.8</td>
<td>306.0</td>
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<tr>
<td>[3] Liquid Arrives</td>
<td>79.550</td>
<td>126.3</td>
<td>235.1</td>
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<tr>
<td>[C] Valve Closes (Shut In Begins)</td>
<td>91.333</td>
<td>126.2</td>
<td>171.4</td>
</tr>
</tbody>
</table>

Increasing

1. Plunger hits Liquid
2. Plunger on Bottom
3. Valve Opens, Unloading Begins
4. Liquid Arrives, Tubing Pressure at Minimum
5. Plunger Arrives, After-flow begins
6. Tubing Pressure Maximum Spike
7. Valve Closes, Cycle Repeats

- Casing Pressure
- Acoustic Signal
- Tubing Pressure
Plunger Controller acts as an on/off switch for Control Motor Valve.

**Valve Closed**

- Elapsed time surface valve is closed (No Sales)
- Flow shut-in as plunger falls to tubing bottom
- Minimum off time or required casing pressure build up

**Open**

Off Time (Length of Shut-in)

- Tubing Pressure: 272.8 psi (g)
- Casing Pressure: 306.0 psi (g)
- Acoustic: 0.078873
Unloading and After-flow Periods

Surface valve open and stored casing pressure expands to lift the accumulated liquid and plunger to the surface.

On Time (Length of Flow)
- Elapsed time valve is open (Gas sold)
- Liquid is Unloaded from well
- Gas is produced and well loads up with next liquid slug

After-flow Gas
Surface valve open, plunger held at surface by differential pressure from flow of gas up the tubing. Well is producing gas. Most liquid produced from the formation tends to fall back, accumulating at the bottom of the tubing.
Shut-in Begins When the Flow Line Motor Valve Closes (Flow down flow line stops)

- Casing Pressure Increases
- Uniformly Spaced Tubing Collar Acoustic Reflections
- Quieter when Plunger on Bottom

Tubing Pressure Increases

Zoom in to 5 minute X-axis Range to see Details
Shut-in Begins When the Flow Line Motor Valve Closes (Flow down flow line stops)

Tubing Pressure drops when plunger starts to fall

Uniformly spaced tubing collar acoustic reflections

Tubing pressure increasing

Pick beginning of shut-in when tubing pressure just starts to increase
Take Guess Work Out of Setting Shut-in Time

- 201 Ft/min Gas
- 38 Ft/min Liquid
- Plunger Hits Liquid
- Plunger on Bottom
- 38 Ft/min Liquid

Fall Velocity in Liquid
- Gassy Fluid: 38 Ft/min
- Surfactant: 80 Ft/min
- High Pressure: 23 Ft/min

Only Shut-in Time Period Shown
Count Signals from Plunger Falling thru Collar: Acoustic/Pressure Signal During Shut-in (1 minute)

1800 Data Points in the Acoustic Signal During 1 Minute
Velocity: Plunger Fall Speed Between Two Consecutive Counted Collars

Plunger Velocity @ Joint 22 equals the change in depth divided by the change in elapsed time.

Velocity = \( \frac{D_i - D_{i-1}}{T_i - T_{i-1}} = -230.9 \text{ ft/min} \)

\[ D_{i-1} = 676.2 \]
\[ D_i = 708.4 \]
\[ T_{i-1} = 5.663 \]
\[ T_i = 5.802 \]

Looking at this Minute Falling through Gas

Each Joint

<table>
<thead>
<tr>
<th>#</th>
<th>Time (mins)</th>
<th>Velocity</th>
<th>Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>C19</td>
<td>5.359</td>
<td>-209.24</td>
<td>611.80</td>
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<td>C20</td>
<td>5.519</td>
<td>-201.95</td>
<td>644.00</td>
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<td>C21</td>
<td>5.663</td>
<td>-223.78</td>
<td>676.20</td>
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<td>5.802</td>
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<td>708.40</td>
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<td>C23</td>
<td>5.961</td>
<td>-203.37</td>
<td>740.60</td>
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<tr>
<td>C24</td>
<td>6.109</td>
<td>-217.08</td>
<td>772.80</td>
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<tr>
<td>C25</td>
<td>6.254</td>
<td>-221.22</td>
<td>805.00</td>
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<tr>
<td>C26</td>
<td>6.411</td>
<td>-205.53</td>
<td>837.20</td>
</tr>
<tr>
<td>C27</td>
<td></td>
<td></td>
<td>369.40</td>
</tr>
<tr>
<td>C28</td>
<td></td>
<td></td>
<td>301.60</td>
</tr>
</tbody>
</table>
Normal Fall Velocity [During Shut-in]

Falling through Gas Gradually Slows from 240 ft/min to 135 ft/min.

Normal Fall Velocity Profile:
1) Tubing is OK
2) Liquid in Bottom

Falling thru Liquid

Click on Any Point
Plunger Fall Velocity Through Gas Free Water

Test Results from Plunger Well Simulator, T-RAM Canada
Dry Gas Pressure Required to Float Plunger at Constant Height at Low Pressure Constant Flow

Test Results from Plunger Well Simulator, T-RAM Canada
Passive Monitoring Requires High Resolution Pressure & Acoustic Data

Signals Show Plunger Falls Past 80th and 81st Tubing Collar

Acoustic Signal

Tubing Pressure - Psig

2568 Ft Deep

0.1 PSI
When Shut-in Begins the Tubing Pressure Drops as Plunger Starts to Fall

Pressure Drop = Weight / Area

Pressure Drop = 2.4 psi

Plunger weight (8 lbs) / Area of 2-3/8"
When Shut-in Begins the Tubing Pressure Instantly Drops when Plunger Starts to Fall

Pressure Drop = Weight / Area

Plunger weight (8 lbs) / Area of 2-3/8”
Identify When Plunger Becomes Stuck
Pressure Increases By ~3 psi

- Blast from Gas Gun Re-Starts Fall
- Plunger Sticks
Paraffin Stops Plunger Fall
9 Shots Used to Push Plunger to Bottom

Pressure Pulse Sent Down Tubing from Gas Gun Applies Force to Top of Plunger to Push & Re-Start Fall

Tubing Pressure Signal Becomes Flat when Plunger Stuck
Chemical Treatment Down Tubing Tends to Slow/ Stops Plunger Fall

Plunger Does Not Reach Bottom...

Fast Plunger Arrivals are a Symptom of Sticking Plunger
Increase in Gas Flow Rate Past Plunger Results in Plunger Slowing Down…

SV Opens

Increase in Gas Flow Rate

Plunger Slows Down
2 3/8” By-Pass Shut-in Period
Notice Pressure and Acoustic Signals

After Flow
Shut-in

Average Fall Velocity
911 Ft/Min

Tight Spot in Tubing Plunger
Sticks at 60th Joint (1866 ft)

Tubing Pressure

3 Psi Step

Acoustic
Fall Velocity – Bypass Valve Closes

Plunger Slowed from 950 ft/min to 366 ft/min due to Valve Premature Closing

@ 3016’ Bypass Valve Closed

By-pass Plungers Just don’t Work in my wells. Why?
2 7/8 inch Bypass Plunger w/ Standing Valve

Hits at Bottom **Very** Hard ~ almost 60 Mile/Hr

Fell 5339 Ft in 1.73 Min 3083 ft/min

Rise Velocity 556 Ft/Min

Gas Velocity = 44.7 ft/sec Near Critical

Gas Flow Rate: 5000 m3/D 176.6 Mscf/D

Line Pressure 30 Psia
2 7/8 inch Bypass Plunger
Fall Velocity Range 5000-1000 ft/min

Fall Velocity Averaged 3654 Ft/Min

Fell 5339 Ft in 1.73 Min
3083 ft/min

60 MPH

Bottom of Tubing - 5339.24 Ft
Plunger Hits Liquid - 5242.71 Ft

Plunger Fall Velocity - Ft/Min
Elapsed Time - M ins
Depth to Plunger - Feet
Tubing Pressure Helps to Identify Downhole Problems

Plunger falls Past Hole at 1872 feet and Pressure from Casing Flows Into Tubing

Rapid Tubing Pressure Increase if Plunger has Sudden Stop

~ 3 psi Drop when Released from Catcher

Hole in Tubing 59th Joint

Suddenly Starts Suddenly Stopped

Plunger’s Weight is Supported by Tubing
Fall Velocity Slows in Deviated Well

Plunger Slowed from 200 ft/min once plunger goes past Kick off Point 8234 Ft

What Effect Does Wellbore Deviation Have on Plunge Fall Velocity?
Horizontal Well Impacts Velocity

What Effect Does Wellbore Deviation Have on Plunger Fall Velocity?

Viper Plunger Fall Slowed Down from 344 to 280 ft/min After going past Kick off Point
Horizontal Well Impacts Velocity

Dual Pad Plunger Increased Speed from 230 to 450 ft/min once Plunger Goes Past Kick Off Point

Solid Plungers Decrease Speed VS Padded Plungers Increase Speed?
Pad Plunger in Deviated S-Curve

- Pad Plunger Looses Seal @ 20 Deg of Inclination
- Speeds Up from 220 ft/min to 360 ft/min
Solid Plunger in Deviated Well

When Inclination > 20 Deg
Extra Friction Slows Down from 500 ft/min to 425 ft/min
Plunger Fall Velocity Determined by Orifice Selection

10 mm
8 mm
4.7 mm
0 mm
Control Fall Velocity with Orifice Size

The graph illustrates the relationship between tubing pressure and fall velocity for different orifice sizes. The y-axis represents fall velocity in feet per minute (Ft/Min), and the x-axis represents tubing pressure average during the fall in psig (pounds per square inch gauge). Four different orifice sizes are shown:

- 8 mm
- 10 mm
- 4.7 mm

The lines for each orifice size represent the fall velocity at various levels of tubing pressure. The orifice size affects the fall velocity, with smaller orifices generally resulting in higher fall velocities at the same pressure levels. The term 'Viper Barstock' might refer to a specific type of tubing or material used in the experiment.
Plunger Fall Velocity Impacted By:

1. Diameter of Plunger – Larger Diameter Falls Slower
2. Effectiveness of Seal between Plunger and Tubing – Better Seal Plunger Falls Slower
3. Brush stiffness – If the Bristles do not provide an effective seal then the plunger falls faster
4. Increased friction due to contact with the tubing – Plunger Falls Slower
5. Old age/increased wear – as the plunger wears out the worn plunger falls faster
6. If Gas can pass through plunger (i.e. Bypass) – then a plunger falls faster
7. When the plunger becomes stuck and stops – usually indicated by a 3 psi increase in pressure
8. If the Tubing is Sticky – the plunger falls slower

Plunger Fall Velocity Impacted By:

9. Wellbore Deviation – more than 20 degrees of deviation impacts plunger fall velocity
   a. Padded Plungers Faster due to Loss of Seal
   b. Solid Plungers Slower due to Increased Friction

10. Gas Flow Rate Into The Tubing – gas flow into tubing reduces plunger fall velocity

11. Pressure or Density of Gas
    a. High Pressure and plunger fall is Slow
    b. Low Pressure and plunger fall is Fast

12. Liquids increase density – plunger falls slow
    a. Surfactant lightens gradient and plunger falls faster, but more time may be required
    b. High pressure also causes plunger to fall more slowly through liquid
Manufacturer Designed Brush Stiffness and Seal Impact Fall Velocity

New Brush Fall Velocity Ranges from 160 - 425 Ft/Min
By-pass and 2 Piece Plungers

1. Plungers with gas bypass ~ fall very fast
   a) Bypass may close
   b) Cylinder may catch ball, before bottom
   c) Heavy Plungers can hit bottom hard and cause damage
   d) Impact of plunger at top/bottom require Liquid to cushion/absorb energy

2. Benefits of Fast Fall against flow
   a) Many cycles per day possible
   b) Able to lift high liquid rates
   c) Maintain Low FBHP (no shut-in required)
1. Data used to correlate construction features of plungers to fall velocity.

2. Some features cause a plunger to fall rapidly, while other features cause a plunger to have a slower fall velocity.

3. Well conditions (gas flow rate and pressure) have significant impact on plunger fall velocity.

4. Use plunger fall velocities to determine shut-in time:
   a. Velocity not accurate
   b. Impacted by many parameters

5. Setting controller to the shortest shut-in time will maximize oil and gas production.
Fall Velocity is Faster at Low Pressures
Slows as Pressure Increases
Plunger Fall Velocity Models

Drag Model: \[ C_d \rho A V^2 / (2g\epsilon) = W_t \]

1. Set Plunger Weight to Drag
2. For a Specific Plunger in a Well a Constant Mass Flows Past the Plunger
3. Kinetic Energy ~ Plunger’s Weight pushes on gas and Velocity Changes to Pass Constant Fluid Mass Past Plunger @ P & T

Orifice Model: \[ (C_d A_{\text{Ann}}) \sqrt{2W_t g \epsilon / (A \rho)} = V A \]

1. Flow Area Between Plunger and Tubing Acts Like a Choke
2. For Specific Plunger the \( dP \) Across Plunger Supports the Weight plus the Choking of Gas Flow Controls the Fall Velocity

\( C_d \) - Drag Coefficient
\( A \) - Area of Tubing
\( V \) - Plunger Fall Velocity
\( W_t \) - Weight Plunger
\( \rho \) - Fluid Density
\( g \epsilon \) - Gravitational Constant
General Model:

\[ V = \frac{C}{\sqrt{\rho}} \]

Compare Orifice Model to Drag Model
Derive the General Model From the Drag Model

Step 1
\[ C_d \times A = W_t \times 2g_c/\left(\rho V^2\right) \]

Step 2
\[ V = \sqrt{\frac{2W_t g_c}{(C_dA)}} \]

Step 3
\[ V = \frac{C}{\sqrt{\rho}} \]

Where
\[ C = \sqrt{2W_t g_c / (C_dA)} \]
1. Input Known Fall Velocity to Calibrate Model
2. For Fall Velocity Calculation Specify Pressure Range
3. Click Calculate Fall Velocity Button to Plot Calculated Fall Velocities
4. Go to Output Worksheet to see Plot
Model Predicted Plunger Fall Velocity @ P & T

\[ V = \frac{C}{\sqrt{\rho}} \]

Use Model Equation to Calculate @ Any Desired P & T:
1) Determine Gas Density – \( \rho \)
2) Plunger Fall Velocity (Ft/Min)

Measured Fall Velocity, ft/min: 300
Average Temperature @ Measured Vel, F: 135
Average Pressure @ Measured Vel, Psia: 261.7
# Measured Fall Velocity in 40 Different Wells

**All Same Type of Cleanout Plungers Installed in Wells**

<table>
<thead>
<tr>
<th>Average Fall Pressure (psi)</th>
<th>Measured Average Fall Velocity (ft/Min)</th>
<th>Model Average Fall Velocity (ft/Min)</th>
<th>Difference Measured - Model (ft/Min)</th>
<th>Model's Absolute % Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>140</td>
<td>296</td>
<td>412.9</td>
<td>-116.9</td>
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<td>216</td>
<td>370</td>
<td>331.0</td>
<td>39.0</td>
<td>10.5</td>
</tr>
</tbody>
</table>

**Avg Error = 0.7 Ft/Min**

**Avg Abs Error = 11.7%**

**Abs Avg Error = 40.1 Ft/Min**
Measured Fall Velocity in 40 Different Wells

Cleanout Plunger Fall Velocity Calculation Based on Gas Density Model at Pressure and Temperature

Model Input:
300 Ft/Min @ 261.7 Psia

Avg Error = 0.7 Ft/Min
Abs Avg Error = 40.7 Ft/Min
Dry Gas ~ Terminal Plunger Fall Velocity Measured in Laboratory: Specific Gas Rate to Float Plunger

Lab Measured Cleanout Plunger Fall Velocity at 17.3 Psia of 1352.5 Ft/Min

Model Input: 300 Ft/Min @ 261.7 Psia
Dry Gas ~ Calculated Terminal Plunger Fall Velocity at Floating Gas Rate

Test Results from Plunger Well Simulator, T-RAM Canada

Sorted by Calculated Plunger Fall Velocity
Faster Cycle: More Production

Fast cycles, smaller slug of liquid, lower buildup pressure required, results in lower average BHP, which results in more production!!

Faster cycles do not mean faster rise velocity in tubing!! But faster fall can be beneficial!
Fall Velocity

- From Know Plunger Fall Velocity Use Model to Predict Fall Velocity at other P & T.
- Acoustic Instrument Is An Effective Method To Measure Fall Velocity and Provide Input Into the Model.
- Changing the Plunger Cycle Impacts Operating Pressure, Model Calculates New Shut-in Time.
- Knowing Fall Velocity Will Ensure That The Plunger Will Reach Bottom By The End Of The Shut-in Period.
Plunger Life Cycle

1. Well is flowing above critical with all flow in Mist flow, no liquid gradient at any time.

2. Well begins to bubble and slug (Usually high speed bypass candidate if +15 ft/s velocity is available.)

3. Well begins to have difficulty maintaining seal due to velocity getting below 15 ft/s (usually good application for padded bypass plunger)

4. Well requires shut-in time to build pressure to maintain velocity of plunger (quick-drop application).

5. Well requires build time (conventional plunger lift applicable as fall time is not important)

6. Well requires substantial build time (high efficiency seals require more fall time but have a better seal).

7. Economics need to be reviewed for rod pump, compression, chamber lift or other forms of lift.
“Select Correct Plunger for the Well” some wells need fast plungers and some wells casing pressure builds slowly.
Plunger Target Rise Velocity

See SPE 14344, *Defining the Characteristics and Performance of Gas-Lift Plungers*


Gas Slips By Plunger
May Never Reach Surface

Wastes Well psi
Damages Equipment

Too Slow

Optimized!

500 to 1000 feet per minute

Too Fast

2. Modified Foss and Gaul model used in this study.

3. Show Impact of Low Line Pressure when Producing Gas Wells

4. Prediction accounts for liquid load, frictional effects, tubular sizes and lengths, surface line pressure and fluid properties.
The Concept of F&G Model: Original

\[ P_1 V_1 = P_2 V_2 \]

Foss and Gaul based on Ideal Gas Law

- \( P_{c, \text{max}} \) when well ready to open
- Casing Volume Only
- \( P_{c, \text{min}}, \) slug and plunger surface
- Casing & Tubing volume

\[ P_1 = P_{c, \text{max}} \] (Pressure Valve Opens)

\[ P_{c, \text{max}} = \frac{V_2}{V_1} P_2 \]

\[ P_{c, \text{max}} = \frac{(A_{\text{csg}} + A_{\text{tbg}})}{(A_{\text{csg}})} \times P_2 \]

\[ P_{c, \text{max}} = \text{CPR} \times P_{c, \text{min}} \]

\[ P_2 = P_{c, \text{min}} \] (Pressure Plunger Arrives)
Equations Used in Derivation


Derivation of Modified Foss and Gaul accounting for production during the rise time and for plunger leakage during the rise time.

\[
\frac{(P_f + T_f)}{T_f} = \frac{M_1}{M_2} \quad \text{Eqn. A9 where } M \text{ denotes mass.}
\]

\[
\frac{(P_{c, \text{max}})(A_D)}{(P_{c, \text{min}})(A_D)} = M_1(1 - \frac{M_s - M_{\text{gas}}_1}{M_2}) \quad \text{Eqn. A10}
\]

where \( M_s, M_{\text{gas}} \) refer to gas produced, leaked past plunger during plunger rise.

\[
A_t = \frac{\rho_s (A_s + A_D)}{\rho_s (A_s + A_D)} \quad \text{Eqn. A11 where } \rho \text{ refers to mass rate of flow.}
\]

\[
K = \frac{(\rho_s + 460)(T_D/12)}{(144/33.3)(S_g)} \left( \frac{1}{2} \times 32.2 \times 144 \times 3600 \right) \quad \text{Eqn. A12 where } T_D \text{ is unit of lbm}
\]

\[
\rho_s = \text{Slag density, g/cm}^3 \quad \text{Eqn. A13}
\]

\[
P_c = \text{plug, pebble can be calculated from:}
\]

\[
P_c = \text{plug, pebble} \times \frac{62.4 \times S_{g} \times (F_{\text{plug}})}{((T_D/12)(2)(32.2)(144)(3600))} \quad \text{Eqn. A14}
\]

This completes the derivation of the original Foss and Gaul in terms of known parameters. If the flowrate of formation produced gas minus the flowrate of gas produced past plunger during plunger rise.

For the following tubing sizes, \( P_{c, \text{max}} \) and \( K \) have the following values:

<table>
<thead>
<tr>
<th>tubing size</th>
<th>( P_{c, \text{max}} )</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 3/8''</td>
<td>25,500</td>
<td>102</td>
</tr>
<tr>
<td>2 7/8''</td>
<td>45,500</td>
<td>166</td>
</tr>
<tr>
<td>3 1/2''</td>
<td>57,000</td>
<td>252</td>
</tr>
</tbody>
</table>

\( A_{\text{oh}} \) is the annulus cross section area between casing and tubing, ft².

\( A_{\text{mb}} \) is the tubing inside cross section area, ft².

\[ A_{\text{oh}} + A_{\text{mb}} \]

\[
\text{All energy comes from expansion of } \phi_{\text{form}}^x, \text{it can be corrected to }
\]

\[
N_{\text{form}}^x = (\rho_s + 460)(T_D/12) \quad \text{Eqn. A15}
\]

\[
N_{\text{form}} = \frac{\rho_s (A_s + A_D)}{\rho_s (A_s + A_D)} \quad \text{Eqn. A16}
\]

\[
N_{\text{form}} = \frac{\rho_s (A_s + A_D)}{\rho_s (A_s + A_D)} \quad \text{Eqn. A17}
\]
**Combined Foss and Gaul and Rule-of-Thumb-Calculator Inputs:**

<table>
<thead>
<tr>
<th>Manual input bbls in tubing, otherwise 0</th>
<th>2.00 bbls</th>
<th>SI Inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input Oilfield Units</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tubing ID, Inches</td>
<td>2.441 in</td>
<td>6.20 cm</td>
</tr>
<tr>
<td>Casing ID, inches</td>
<td>4.09</td>
<td>10.39 cm</td>
</tr>
<tr>
<td>Average Well Temp, F</td>
<td>130.00 F</td>
<td>54.44 cm</td>
</tr>
<tr>
<td>Pcsq, psi</td>
<td>1066.62 psig</td>
<td>7353.28 kpa</td>
</tr>
<tr>
<td>Ptgt, psi</td>
<td>916.92 psig</td>
<td>6321.25 kpa</td>
</tr>
<tr>
<td>Pp, psi to lift plunger wt, lbs</td>
<td>5.00 psig</td>
<td>34.47 kpa</td>
</tr>
<tr>
<td>Line Pressure, psi</td>
<td>400.00 psig</td>
<td>2757.60 kpa</td>
</tr>
<tr>
<td>Liquid SG</td>
<td>1.00 dimless</td>
<td>1.00 dimless</td>
</tr>
<tr>
<td>Gas Gravity</td>
<td>0.65 dimless</td>
<td>0.65 dimless</td>
</tr>
<tr>
<td>Plgr Fall Vel in Gas, ft/min</td>
<td>250.00 ft/min</td>
<td>76.20 m/min</td>
</tr>
<tr>
<td>Plgr Fall Vel in Liq, ft/min</td>
<td>40.00 ft/min</td>
<td>12.19 m/min</td>
</tr>
<tr>
<td>Depth to Spring, ft</td>
<td>10000.00 ft</td>
<td>3048.00 m</td>
</tr>
<tr>
<td>Fraction of gas in Slug (~0.8)</td>
<td>0.80 Fraction</td>
<td>0.80 Fraction</td>
</tr>
<tr>
<td>Fudge Factor: Adjust Shut-in Time (&gt;1)</td>
<td>1.10 F Factor</td>
<td>1.10 F Factor</td>
</tr>
</tbody>
</table>

**Desired Liquid Production, Rate**

| Desired Liquid Production, Rate | 24.95 bpd | 3.967 m^3/day |

**Plunger Liquid Removal Effcy (per cycle)**

| Plunger Liquid Removal Effcy (per cycle) | 0.90 dimless |
# Combined Foss and Gaul and Rule-of-Thumb-Calculator Results:

<table>
<thead>
<tr>
<th>RESULTS</th>
<th>Oilfield Results</th>
<th>SI Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>bbls in tubing</td>
<td>2.000 bbls</td>
<td>0.318 m^3</td>
</tr>
<tr>
<td>Height of Gassy Liq, ft</td>
<td>1728.65 ft</td>
<td>526.89 m</td>
</tr>
<tr>
<td>Fall time thru gas, min</td>
<td>33.09 min</td>
<td>33.09 min</td>
</tr>
<tr>
<td>Fall time through gassy liquid, min</td>
<td>43.22 min</td>
<td>43.22 min</td>
</tr>
<tr>
<td>Total fall time, min</td>
<td>76.30 min</td>
<td>76.30 min</td>
</tr>
<tr>
<td>Total Fall Time x F Factor</td>
<td>83.93 min</td>
<td>83.93 min</td>
</tr>
<tr>
<td>Csg P to lift at 750 fpm</td>
<td>1066.62 psig</td>
<td>7353.29 kpa</td>
</tr>
<tr>
<td>&amp; Corresponding Tubing P, psi</td>
<td>916.92 psig</td>
<td>6321.25 kpa</td>
</tr>
<tr>
<td>Min Plunger Arrival Time (&gt;1000 Ft/Min)</td>
<td>10.00 min</td>
<td>10.00 min</td>
</tr>
<tr>
<td>Max Plunger Arrival Time (&lt;500 Ft/Min)</td>
<td>20.00 min</td>
<td>20.00 min</td>
</tr>
<tr>
<td># Cycles/Day to Remove Desired Liquid</td>
<td>13.86 cy/day</td>
<td>13.86 cy/day</td>
</tr>
<tr>
<td>Max # Cycles/Day - Using Cycle Time</td>
<td>13.86 cy/day</td>
<td>13.86 cy/day</td>
</tr>
<tr>
<td>Max Possible Liquid Production, Rate</td>
<td>24.94 bpd</td>
<td>3.966 m^3/day</td>
</tr>
<tr>
<td>Minutes per Cycle</td>
<td>103.93 min</td>
<td>103.93 min</td>
</tr>
<tr>
<td>Maximum Unloading Time, Min</td>
<td>20.00 min</td>
<td>20.00 min</td>
</tr>
<tr>
<td>Maximum Afterflow Time, Min</td>
<td>0.00 min</td>
<td>0.00 min</td>
</tr>
<tr>
<td>Minimum Required Gas Rate</td>
<td>99.76 Mscf/D</td>
<td>2824.8 m^3/day</td>
</tr>
<tr>
<td>Desired Plunger Rise Velocity</td>
<td>750 Ft/min</td>
<td>228.6 m/min</td>
</tr>
<tr>
<td>Produced Gas (Formation or Daily Rate)</td>
<td>21 Mscf/d</td>
<td>594.7 m^3/day</td>
</tr>
<tr>
<td>Gas Leaks Past Plunger</td>
<td>21 Mscf/d</td>
<td>594.7 m^3/day</td>
</tr>
<tr>
<td>Csg P @ Input Rise Velocity</td>
<td>1065.45 psig</td>
<td>7345.21 kpa</td>
</tr>
</tbody>
</table>
Questions?